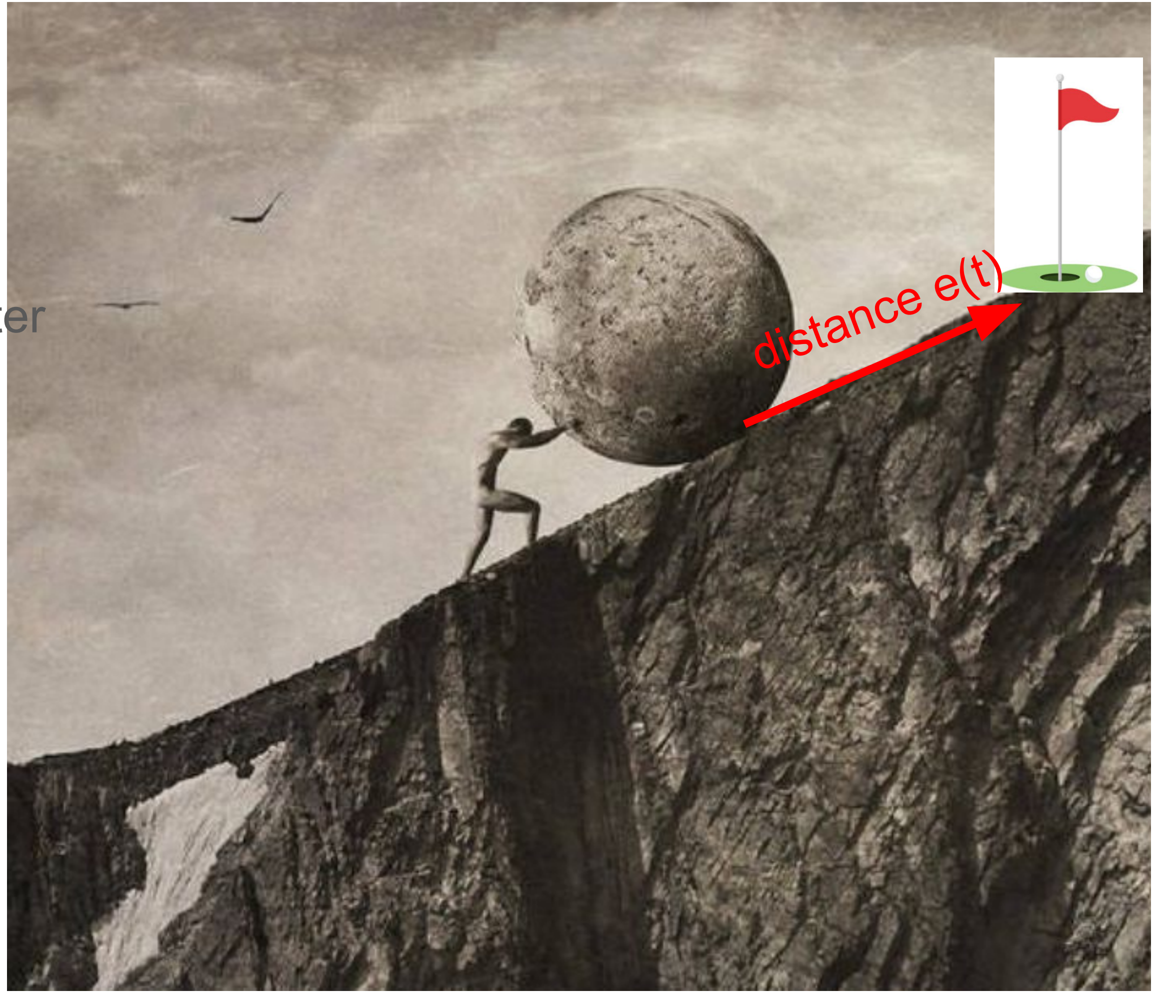
Crash Course on HVAControl Theory by Ben

(Ben’s a math guy)

Explain the game's intuition:

Bread and butter of data:
Excel

(R / Python more powerful, but overkill)



Bread and butter of controls:
PID

(State-Space model more powerful, but overkill)

Explain the game's intuition:

You are the caddy to a blindfolded golfer!
What's your strategy?

Options for strategic guidance:

Golf Clubs:

Tee: Driver
Fairway: Woods, Irons, Wedges
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Controls:

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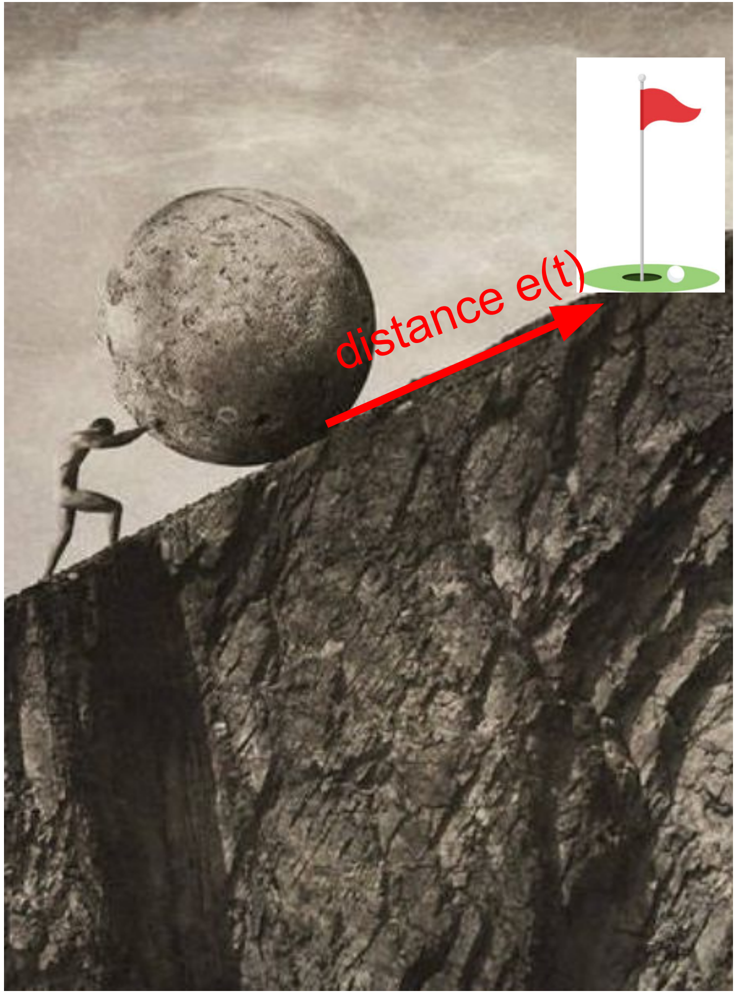

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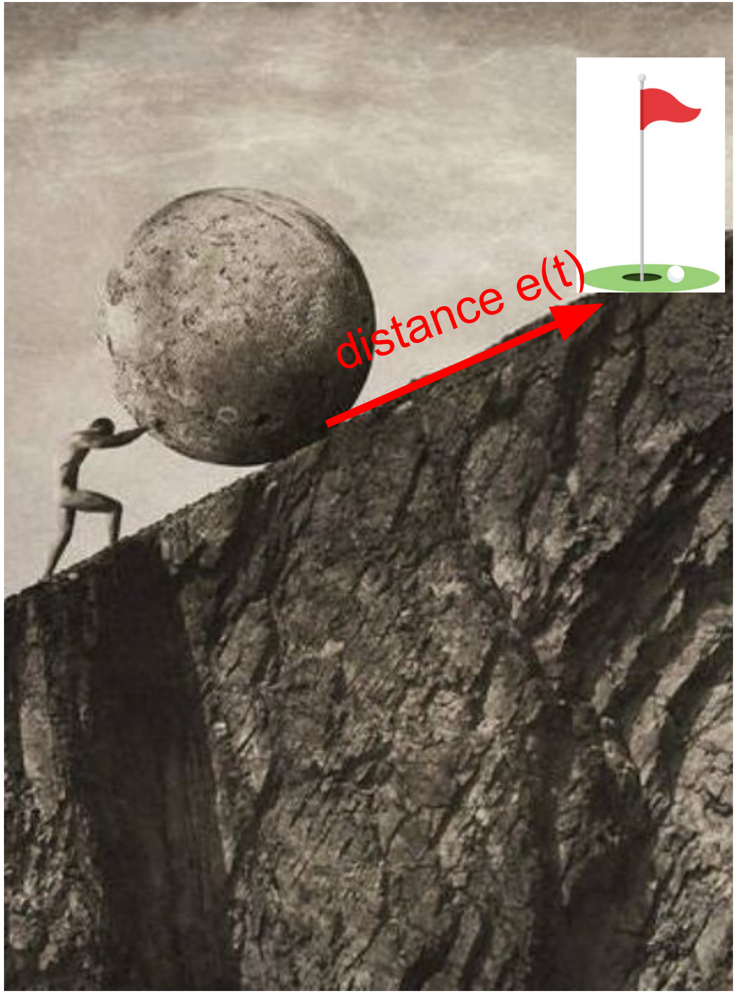

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Loose intuition:

P: Proportional → $e(t)$ → "Take a BIG swing!"

I: Integral → $\int_0^t e(\tau)d\tau$ → "FYI you've been stuck in sand trap a while"

D: Derivative → $\frac{de(t)}{dt}$ → "Hey someone's moving the flag back"

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Only accessible present information! → Decision!

HVAC in Particular:

Asymmetry: Ball rolls down, Sisyphus stuck pushing below

- heater + no AC = passive cooling only

Big Inertia: Sisyphus's gigantic boulder

- Long while for room to heat up
- Slow vent airflow
- Humidity takes time to alter

No Momentum: everything submerged in molasses

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Hints at the following surefire strategy:
simply make P large, then tweak I to correct any long-term offset biases
(intuitively, think about why this should work indeed!)

PI~~D~~

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Panacea PID game's abstract mathematical form

plant/system dynamics: $$\dot{\mathbf{x}} = f(\mathbf{x}, u, t)$$

f and h "nicely behaved" black-box:

sensor/actuator readoutput: $$y(t) = h(\mathbf{x}(t), t)$$

Goal: make tracking error zero $$e(t) = r(t) - y(t)$$

desired reference value

Controller's algebraic form:

$$u(t) = \underline{K_P} e(t) + \underline{K_I} \int_0^t e(\tau)d\tau + \underline{K_d} \frac{de(t)}{dt}$$

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That's ok people still do deep learning + hyperparameter tweaking upon blackboxes!

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f and h “nicely behaved” black-box:

Irony: lack of underlying assumptions behind an unknown black-box makes this the most generalized universal principle and tool

And with simply just the premise of “well-behaved” mathematical functions:

Canonical State Space Reformulation Generalization

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$$

$$\mathbf{y} = C\mathbf{x} + D\mathbf{u}$$

Notion of “well-behavedness” pertinent to PID:

- Exists a localized linear approximation for underlying LTI system
- Convenient framework for mathematical analysis
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While there are fancy general PID tuning methods such as:

- Cohen-coon
- Ziegler-Nichols
- Astrom-Hagglund
- Tyreus Luyben
- SIMC

But for HVAC where even the D in PID is mostly irrelevant, that brings us back to:

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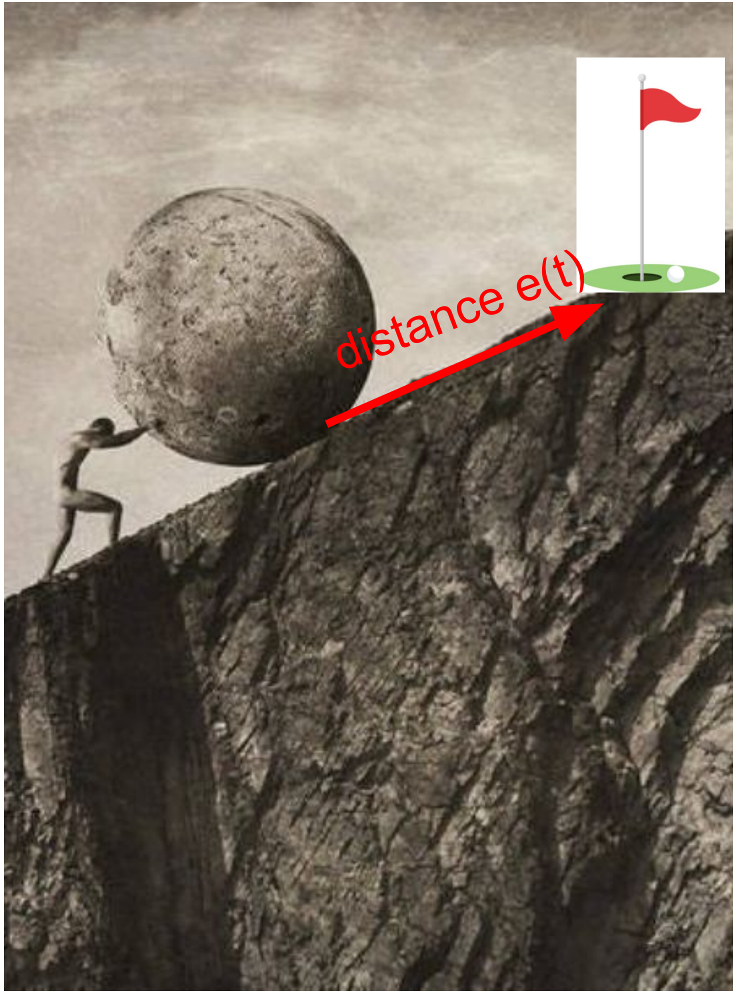

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On a fun brief interdisciplinary note:

They use similar techniques in other fields too:

Economics, for controlling dynamics of economic variables:
Consumption, Production, Interest Rate, Inflation, GDP, etc.

Machine Learning, for time-series-type data:
Inferring control parameters governing dynamics as opposed to classical probability distribution parametric estimation of underlying feature vectors

Physics, for field theory:
Sensitivity analysis of parameters that control a system is the essence of theoretical physics (Hamiltonian Mechanics, Statistical Mechanics, Principle of Least Action, Chaos)

Questions?

Thanks!